THE ALGEBRAIC THINKING PROJECT

Utilizing research and new technologies to overcome the algebra barrier

Fund for the Improvement of Postsecondary Education

Comprehensive Program Proposal

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Invitational Priority #1: Centers of excellence for teacher preparation

THE ALGEBRAIC THINKING PROJECT

A. NEED FOR THE PROJECT

The *Algebraic Thinking Project* will restructure preservice math teacher education by using research on students' thinking to create a *Center for Algebraic Thinking* that will empower new teachers to open the algebra gateway with their students. Algebra is the "gatekeeper" to higher education and future employment because, rather than helping students develop mathematical competence and gain access to higher education, it screens out many students (Ladson-Billings, 1998; Moses & Cobb, 2001). For instance, in the Los Angeles School District, "48,000 ninth-graders took beginning Algebra; 44% flunked, nearly twice the failure rate as in English… Among those who repeated the class in the spring, nearly three-quarters flunked again…It triggers dropouts more than any single subject" (Helfand, 2006, p. 1). "In Grand Rapids Public Schools, nearly 40 percent of algebra 1 students – 1,196 of 3,149 – received a failing or "held' grade. And 22 percent of the 2,101 algebra II students also did not pass." (Murray, 2010). Responding to Chicago's increasing Algebra failure rate, a school official said "It's not surprising that you're going to see an increase in [failure] rates [in Algebra] if you raise the instructional requirements and you don't raise supports" (Viadero, 2009, ¶8). In Oregon, 46% of students in grade 10 failed to meet the state math standards in 2008-09 (Oregon Department of Education (ODE), 2009). This is an especially critical time for Oregon as we are increasing graduation requirements in math so students are more college and career ready. By 2014, all students must not only pass Algebra I, but understand it well enough to take two more advanced math courses in order to graduate high school. The statistics above indicate that teachers are not being adequately prepared to facilitate students' success in algebra. With new graduation requirements, this could create an increased drop out rate unless we re-envision teacher preparation programs across the country to enable future mathematics teachers to stem the tide of failure in algebra.

Many school districts are responding to the algebra crisis by creating two or three-year algebra programs or having students take double periods of algebra, but the student success rate is not changing significantly (Nomi & Allensworth, 2009). It is becoming apparent that the amount of time spent studying algebra is not the issue. We cannot prepare future teachers to teach algebra the same way and expect different results. Institutions currently working on the algebra issue include the Connected Mathematics Program, College Preparatory Mathematics Program, and West Ed. There is some evidence that new curricula from projects like these are improving students' success with algebra (i.e. Moseley & Brenner, 2009; Riordan & Noyce, 2001), but many students still struggle to understand and there is controversy over reform math's effects (i.e. Milgram, 1999). The National Mathematics Advisory Panel (2008) recommends "teachers should understand how students learn to solve equations and word problems and causes of common errors and conceptual misunderstandings. This training will better prepare them for dealing with the deficiencies students bring to the classroom and for anticipating and responding to procedural and conceptual errors during instruction" (p. 4-88). However, no effective resource currently exists for preservice teachers that would easily allow them to understand 'common errors and conceptual understandings' in algebra. A new, innovative approach to preparing future teachers using such a resource is needed that takes into account ***why*** students struggle in algebra.

The literature indicates that students' comprehension of algebraic ideas is often not what it seems. "An error is not merely a failure by a student but rather a symptom of the nature of the conceptions which underlie his/her mathematical activity" (Balachef, 1984). Errors can result from simple carelessness or forgotten rules, such as 5 + 3 = 9. Researchers also find that "some kinds of errors are widespread among students of different ages, independent of the course of their previous learning of algebra" (Demby, 1997, p. 48). These studies indicate common struggles with more complex sources than simple calculation or memory errors. If preservice teachers do not address those underlying ways of thinking and align pedagogy with a student's needs, the "symptoms" will often continue to pop up as the student continues to apply the incorrect rule or variations of that rule in new situations.

Research has documented students' mental hurdles in making the transition from arithmetic to algebra (Booth, 1984; MacGregor & Stacey, 1997; and Moseley & Brenner, 2009). Over 600 articles spanning the last three decades examine why students struggle in algebra[1]. This potentially indispensable resource for preservice teachers remains unusable and largely invisible because of its overwhelming size. While preservice teachers in education programs typically read research articles in Math Methods courses, they, and their instructors cannot be expected to process the entire research base for something useful to their teaching. Math teacher preparation needs a readily accessible, easy to use resource that distills and capitalizes on research that can significantly improve the effectiveness of preservice teachers and increase the success of middle and high school students learning algebra. We will create the innovative *Center for Algebraic Thinking*, a national resource for teacher education institutions that will include an organized taxonomy of the literature and searchable, "encyclopedia" of students' algebraic thinking. Math Methods courses will use this center as an instructional tool to orient preservice teachers towards the role students' thinking should play in their instruction. It will also be a resource for just-in-time understanding of the characteristics of and mathematical issues behind students' struggles

[1] See http://www.willamette.edu/~srhine/Rhine/ATP_References.html

that preservice teachers can use during student teaching and throughout their career.

In addition, formative assessment problems used in this research base are designed to elicit students' range of algebraic thinking and misconceptions of algebraic concepts. This research usually includes percentages of students that respond with correct or incorrect answers, giving preservice teachers an idea of what to expect in typical algebra instruction. These catalogued formative assessments, accessed by preservice teachers from the *Center* or handheld technology, can facilitate their understanding of why particular algebraic ideas are challenging for their students. Studies show that technology based formative assessment tools such as Classroom Response Systems can increase student participation and reshape teacher discourse patterns (Feldman & Capobianco, 2008; Langman & Fies, 2009). Therefore, an important component of *ATP* will be technology based formative assessments.

Studies on student thinking in algebra often include instructional strategies, particularly technology based practices, that effectively address students' misconceptions or struggles to conceptually understand algebraic ideas. Technology has unique capabilities that can help preservice teachers facilitate their students' transition from arithmetic to algebra, but preservice teachers need to be prepared to use these tools effectively. However, in the words of Goldenberg (2000), "Not everything that *can* be done *should* be done" (p. 1). Rather, preservice teachers should learn a "few good tools well enough to use them knowledgeably, intelligently, mathematically, confidently, and appropriately in solving otherwise difficult problems" (p. 7). We have chosen specific technological tools (graphing calculators, spreadsheets, science sensors, and virtual manipulatives) for the *Algebraic Thinking Project* because the literature suggests ways they can be powerful allies in helping students develop conceptual understanding in algebra (i.e. Calder, Brown, Hanley, & Darby, 2006; Neiss, Sadri, & Kwangho, 2008).

Graphing calculators can lead to better understanding of functions and graphing (Hollar & Norwood, 1999) and improve students' ability to connect multiple representations of algebraic concepts (Graham & Thomas, 2000; Herman, 2007). Spreadsheets can act as a bridge between arithmetic and algebra by helping students generalize patterns, develop an understanding of variable, and facilitate transformation of algebraic expressions (Haspekian 2005; Tabach, Hershkowitz, & Arcavi, 2008). Science sensors can increase students' conceptual understanding of math (Lapp & Cyrus, 2000; Mokros & Tinker, 1987). Virtual manipulatives, in particular, (such as those from our partner shodor.org/interactivate) have great potential for facilitating significant gains in students' achievement (Moyer, Niezgoda, & Stanley, 2005; Suh, 2005) and enhancing students' conceptual understanding in algebra (Suh & Moyer, 2007). For example, the literature shows that students can often calculate slopes and write equations for lines without understanding the concept of slope (Zaslavsky, Sela, & Leron, 2002). Shodor's *"Slope Slider"* allows students to manipulate a linear function of the form $f(x)=mx+b$ using slider bars for "*m*" and "*b*" (see Figure 1, Appendix A). Using this applet, students can explore the relationship between slope and intercept by rapidly changing values for "*m*" and "*b*" and simultaneously watching the dynamic between equation and graph. Students explore the meaning of slope and *y*-intercept rather than grind out a graph of line after line on paper, losing meaning along the way. These virtual manipulatives are available through the web, but access to computer labs is often limited and some school districts are hesitant to give students access to the Internet. Furthermore, these manipulatives can be extremely useful for working on specific concepts, but perhaps for only 10 minutes at a time, not justifying the hassle for a teacher to arrange a computer lab. Accordingly, based on needs identified from our review of the research on algebraic thinking, the *Algebraic Thinking Project* will create additional virtual manipulatives accessible to handhelds.

Due to recent advances in handheld technology, all the capabilities of a graphing calculator, spreadsheet, virtual manipulatives, science probes, and Classroom Response System now converge on single handheld devices such as the iPod touch, smart phones, and tablets such as the iPad and HP's Slate. The primary differences between handheld devices and tablets are cost (handhelds are half the cost of tablets) and screen size (tablets are easier for text input and reading e-textbooks), so school districts will choose different resources based on their needs. Publishers and districts are moving towards digitized textbooks (Dybwad, 2010). "Mobile technologies provide an opportunity for a fundamental change in education away from occasional use of a computer in a lab towards more embedded use in the classroom" (Naismith**,** Lonsdale, Vavoula, & Sharples, 2004, p. 6). Our revised courses will focus on how to use this technology pedagogically to address issues raised by research on student learning in algebra.

The consequence of *ATP* at four public and private universities in Oregon (George Fox University, Pacific University, Western Oregon University, and Willamette University), will be restructured Mathematics Methods courses that lead to increased preservice teachers' knowledge, skills, and dispositions towards students' algebraic thinking. Preservice teachers' students will ultimately have better algebra understanding, higher test scores, graduation rates, and college enrollment. Nationally, our innovative web-based resources and experience implementing the courses in multiple contexts will facilitate easy replicability across the country.

B. SIGNIFICANCE OF THE PROJECT

Our restructured Mathematics Methods courses are significant because they will develop effective preservice teachers who can respond to students struggling with algebra by using the *Center for Algebraic Thinking* website to:

- learn from research *why* students struggle with that particular concept in algebra,

- find formative assessment tools that will help them assess the depth and breadth of students' understanding,
- learn technological tools that help develop students' conceptual understanding of a topic, &
- discuss their experiences regarding students' algebraic thinking and relevance for teaching.

The *Algebraic Thinking Project* is based on evidence from research that teacher preparation which focuses upon developing teachers' understanding of students' thinking (Carpenter, Fennema, Franke, Levi, & Empson, 2000; Gearhart, et. al, 1999), data-driven instructional decision making (Darling-Hammond, et. al, 2009), and the value of technological tools for developing conceptual understanding (Battista & VanAuken Borrow, 1998: Rojano & Sutherland, 1993; Tabach, Hershkowitz, & Arcavi, 2008) is likely to improve student achievement in algebra. The models for our restructured mathematics teacher preparation, Cognitively Guided Instruction (CGI) from the University of Wisconsin, Madison[2] and Integrating Mathematics Assessment (IMA) from UCLA demonstrated that when elementary teachers understood why students' struggle and have resources to develop that understanding then they were more likely to adjust instruction in ways that meets students' needs. Teachers took on a new orientation towards assessing and understanding students' thinking. Their beliefs and practices changed in ways that helped them facilitate increases in student achievement.

A significant aspect of the *ATP* is modifying the CGI and IMA models for teacher preparation for middle and high school preservice teachers so they develop dispositions towards using research on student thinking as central to their teaching practice. Once preservice teachers have this orientation, the *Center* will become a resource for their student teaching and their careers as just-in-time professional development, as they can continually return to the *Center* to

[2]For information about CGI, see http://www.promisingpractices.net/program.asp?programid=114#programinfo

learn about students' thinking and technology based instructional approaches that are relevant to the content they are teaching. Furthermore, *ATP* will model the value of a forum in the *Center* in which preservice teachers in the field can engage in dialogue with other preservice teachers and Mathematics Methods faculty across our university campuses about students' thinking and learning to teach with that thinking in mind. The forum will grow to include inservice teachers and teacher education faculty and students across the country.

ATP's **local** significance will be immediate impact on our universities' preservice teachers' knowledge, skills, and disposition teaching algebra, resulting in increased pass rates for middle and high school students during student teaching in Algebra classes. Because algebra is the basis of higher mathematics, long term impact will likely include preservice teachers' increased confidence in the classroom and increased students' success on state assessments in math, decreased dropout rates, increased success and enrollment in higher math, and increased high school graduation rates and college enrollment. We will model cooperation across our campuses that will facilitate continual renewal and effective use of the resources at the *Center.* Combined, our universities prepare over 100 future middle and high school mathematics teachers annually.

The **national** significance of the *ATP* will be our model of mathematics teacher preparation that can be replicated at other institutions. Significance will include processing and organizing hundreds of research studies so they are accessible to and usable by teacher education programs as well as inservice teachers across the country. The *Center* will be a national model for future centers that could focus on other well-researched areas in mathematics thinking such as geometry, statistics, and probability. The web-based resources of the *Center* would be available to other teacher preparation institutions as well as school districts beginning summer of the second year of the grant and be applicable regardless of the curriculum used in a district. We will demonstrate the effectiveness of our resources and model and provide professional consultation for other institutions that want to use the *Center* and replicate our Mathematics Methods courses.

C. QUALITY OF THE PROJECT DESIGN

The *Algebraic Thinking Project's* (*ATP*) purpose is to develop innovative Mathematics Methods courses based on research into students' algebraic thinking and the resources of the *Center for Algebraic Thinking.* As a result, *ATP* preservice teachers will address core algebra content in their field experiences more effectively (see Figure 2, Appendix A for a preservice teachers' instructional processes with *ATP*). The CGI and IMA programs are the models for our restructuring. Both programs used research-based knowledge of students' mathematical thinking, studied teachers' knowledge of students' mathematical thinking, and then facilitated the way teachers might use knowledge of students' mathematical thinking in their assessments and instruction. We will base the primary features of our courses on the successful examples of CGI and IMA and extend their model to middle and high school algebra by:

1) Engaging preservice teachers in conversations about their intuitive knowledge about students' algebraic thinking in a particular content domain.
2) Building on that knowledge by introducing preservice teachers to relevant research.
3) Facilitating preservice teachers' creating frameworks regarding students' thinking.
4) Facilitating preservice teachers using this knowledge to develop their practice.

Year 1: Research Synthesis, Preservice Teacher Study, and Resource Development. Oregon is phasing in the Common Core State Standards over the next few years. Therefore, we will synthesize the research with both sets of standards in mind, organizing the algebra research studies into five cognitive domains--variables, algebraic relationships, analysis of change, patterns and functions, and math modeling--to guide our work (see Table 1, Appendix A). The project will initiate with math professors, math educators, and practicing algebra teachers combining to make five teams of three, each synthesizing the research of one algebraic cognitive domain. Each Domain Team will determine a chosen segment of the literature to read monthly over the year and meet one day each month to collaborate on synthesis of the research. These studies typically include a discussion of the math involved in a misconception or way of thinking about algebra, assessment problems, transcripts of interviews with students describing their thinking with problems, and suggested instructional strategies to address misconceptions or ways to develop understanding. Accordingly, five questions will guide the synthesis of research:

1. What Oregon and Common Core State Standard(s) does this research address?
2. What is the symbolic representation of thinking with the idea? (What does it look like?)
3. How do students think about the algebraic idea? (What does it sound like?)
4. What are the underlying mathematical issues involved?
5. What research-based strategies/tools could a teacher use to help students understand?

In spring of Year 1, all Domain Teams will gather for one-week to design a coherent and well-organized taxonomy of the research synthesized, including a formative assessment problem catalog and affordances of existing tools that facilitate development of algebraic thinking (see Project Timeline Chart, Appendix A). Domain teams will also identify assessment problems from research so that in spring, they can design a catalog of formative assessments of students' algebraic thinking. In spring, teams will also design a handheld technology application so preservice teachers can use these assessments in class and get data about each student's understanding instantaneously. Preservice teachers can use assessments without technology as well, although they won't have the advantages of instantaneous and anonymous feedback about a class' understanding.

Lastly, much of the literature refers to technology as a tool for developing students' conceptual understanding in algebra. These tools include graphing calculators, spreadsheets, virtual manipulatives, and science probes. Through the fall, Domain Teams will document how research indicates each tool can facilitate understanding of specific topics in algebra. In cases in which no technology currently exists to address an algebraic concept, when appropriate, domain teams will design virtual manipulatives or new uses of these technologies that would facilitate developing conceptual understanding. In spring and summer of Year 1, we will design, create, field test, and refine 5-10 new virtual manipulatives, or 1-2 per cognitive domain. These will supplement a suite of algebra applications including formative assessment tools and access to current technologies such as a graphing calculator, spreadsheets, and science probes for handheld and tablet devices. The team at Shodor will program new virtual manipulatives. The team at Willamette University will focus on an application to deliver formative assessments and also develop new manipulatives. The team at Vernier will adapt science probe interfaces for handhelds and tablets. In fall, the Director will recruit two middle and two high school teachers to do field testing of all *ATP* resources as they are developed during the year. Teachers will explore with their students unexpected software issues, bugs in the programs, and general usability of the applications. Based on teacher reports and informal observations from the field tests, the Core Team[3] will work with the Programming Teams to refine the applications.

Beginning in late fall of Year 1 we will collect video of students' thinking using the formative assessment problems identified from research. An important feature of our restructured courses will be videotaped interviews of students explaining their thinking when engaged with assessment problems. In CGI and in Dr. Rhine's experience with IMA, the most powerful tool

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[3] Drs. Rhine, Samek, Charles, Harrington, and Starr.

for teachers' development was the opportunity to listen to students explaining the range of their thinking. These videos will also model interviewing strategies for preservice teachers to consider for their instruction. The *ATP* Director will choose students from field test teachers' classrooms based on students' performance on the formative assessments. As there are over one hundred of these problems in the literature, the domain teams will prioritize which problems to videotape that they believe would be most beneficial for teachers. We will continue to capture video of students' algebraic thinking each semester throughout the first two years of the project. The Director, who collected these types of interviews during IMA, will edit student videos with work study support. We will videotape 2-3 different students per formative assessment problem and 3-5 formative assessment problems per cognitive domain.

CGI and IMA studied teachers' knowledge of students' math thinking and the way teachers might use knowledge of students' thinking in making instructional decisions. In Year 1, the *ATP* Core Team will similarly examine preservice teachers' intuitive and experiential knowledge of students' algebraic thinking in middle and high schools through surveys and interviews of preservice teachers in Mathematics Methods courses. The Domain Teams will compare what is known from the literature to what preservice teachers know about students' algebraic thinking. This identified gap will become the initial focus of our courses.

Based on the synthesized research, formative assessment catalogue, instructional technology, and videotape of students' algebraic thinking we will create course modules for preservice teachers. These hour-long, web-based modules will include:

- insights from research on students' thinking in an algebraic cognitive domain,
- video clips of students' describing their thinking as they encounter the formative assessment problems,

- activities that develop teachers' content knowledge in algebra,
- demonstration of an instructional technology appropriate for developing students' conceptual understanding of the topic.

We will begin designing modules in fall of Year 1 and creating them in the spring. Finally, at the end of spring in Year 1, we will design the *Center* website as a delivery vehicle for the modules, online encyclopedia of algebraic thinking, catalog of formative assessment problems, and matrix of instructional technology, organized around the five algebraic cognitive domains. Programming of the *Center* website will be contracted out.

Year 1, Summer: Course Design. We will devote summer of Year 1 to designing the Mathematics Methods courses. With hundreds of research studies on algebraic thinking, the purpose of the course is not to lecture to preservice teachers on all of the information from that research. Rather it is to develop teachers' disposition towards timely use of *ATP* resources to assess and use students' thinking in their instructional decision making. As with CGI and IMA, we will begin the course with exploration of preservice teachers' current content knowledge and intuitive knowledge of students' algebraic thinking in that domain—thereby modeling the approach we hope teachers will take with their students. We will watch video clips of students describing their thinking and engage in activities that help teachers encounter the types of struggles that students face (See Rhine & Bennett, 1998, for a description of those types of activities). Then we will discuss research-based formative assessments and the features of assessments that effectively evaluate the depth and breadth of students' understanding of algebra. We will also introduce teachers to the *Center* website as a resource for our courses and for their continued use during student teaching. Our work with methods courses will have implications for Planning and Assessment courses in teacher education and supervision of student teaching.

Accordingly, the Core Team will meet with those faculty at each university to identify ways to change the content of those courses to support the goals of our restructured Methods courses.

As a result of our courses, we expect preservice teachers to have a greater breadth and depth of understanding of students' algebraic thinking. Using that insight, preservice teachers will be able to make more effective decisions regarding instructional activity that will improve the learning trajectories of their students. Those decisions will include consideration of technological tools that can support development of students' conceptual understanding. We will design activities that will help preservice teachers learn to integrate technological tools into their instruction that facilitate the development of algebraic concepts identified in research as challenging for students to learn. During the spring, the Director will organize field tests of the new virtual manipulatives created in the fall and as more are created during the year.

Year 2: Course Pilot. Each of our universities' Mathematics Methods courses are structured slightly differently regarding length (semester or year long) and coordination with student teaching. Accordingly, we will implement the restructured courses at different points and in different ways during the year. This experience will be informative for other teacher education institutions in the country that may have similar structures. During the pilot and then implementation of the Mathematics Methods courses in Year 3, the Core Team will conduct case studies with eight preservice teachers in student teaching (two per university), analyzing preservice teacher reflection logs, university and site supervisors observing in classrooms, and interviewing preservice teachers regarding use of the resources twice a month. The Core Team will meet monthly to review evaluation data on an ongoing basis. Based on the data, we will revise the *Center* resources and design of the Mathematics Methods courses. Each university will have a technology library consisting of two sets of 15 handhelds and two sets of 15 tablets for preservice teachers to take into their student teaching experiences if the technology is not available at their site. This will create three contexts for implementation of the *Center* resources for preservice teachers to experience: handheld, tablet, and computer lab access.

Year 3: Implementation, Refine Courses and Resources. By summer, Year 2 we will complete the development of *ATP* resources. Based on the data collected from case studies, observations, surveys, and assessments, we will refine our courses, modules, formative assessments, web site, and technological tools, in preparation for implementation in Year 3. In Year 3, we will implement the courses as intended and assess the impact on our preservice teachers and their students. In summer following Year 3, we will complete refinement of our courses, teaching modules, and virtual manipulatives. The *ATP* Core Team will analyze data and our external evaluator, the Research Division of the International Society for Technology Education (ISTE), will complete its report so that we can determine findings on the potential impact of the *Algebraic Thinking Project* on preservice teachers and students.

Sustainability and Dissemination. The resources of this grant will establish the *Center for Algebraic Thinking* and all its resources as well as a model for using the resources in Mathematics Methods courses in teacher education programs. In order to sustain the work of the grant, and continue to be an effective source of professional development, a system of renewal and refinement for the intellectual resources of the *ATP* will be in place. We estimate that clerical work for the *Center* of approximately 5 hours per month are needed to acquire newly published research on algebraic thinking and identify advancements in technology. Willamette University will commit to funding those hours. Also, after the initial purchase with grant funds, each university will sustain the tech libraries with a lab fee to students that will continually renew the library. As Director of the *Center*, Dr. Rhine will convene a small group of math professors, teacher educators, and math teachers once a month to review new research and tools and create new algebraic thinking modules or refine old ones. In Oregon, there is an annual meeting of the organization Teachers of Teachers of Mathematics (TOTOM) in which most teacher education institutions in the state are represented. Throughout the three years of the grant and beyond we will disseminate and discuss our experiences and resources through TOTOM so that other institutions in the state can replicate our program.

Nationally, the Core Team will disseminate our resources and experiences in using them at various conferences beginning in Year 3. The *ATP* Core Team will submit papers to peer-reviewed journals and present at various conferences in order to engage in discussion regarding the features, challenges, and success of our *Center* and model for teacher education. Willamette University will host a national conference in Fall, 2014 focusing on strategies for teacher preparation institutions to improve the success of preservice teachers of students in algebra courses. At each of those conferences, the Core Team will actively seek institutions that are willing to commit to using the *Center* and replicate our model of math teacher preparation. With a small charge to users of the *Center* resources, we will be able to expand capacity to facilitate replication of the project as interest in the *Center* grows so that clerical and Core Staff will be compensated for their time after the grant funding concludes.

D. QUALITY OF PROJECT EVALUATION (see Project Evaluation Chart, Appendix A)

Objectives. By the close of the project, the ATP will deliver three interrelated components:

• The *Center for Algebraic Thinking* website to support Math Methods courses, including:

-a research synthesis and taxonomy of over 600 studies on algebraic thinking, organized into five cognitive domains of the Oregon and Common Core State standards and searchable as an encyclopedia of algebraic thinking

-a catalogue of a minimum of 50 formative assessment problems

-a matrix identifying appropriate technology to help students comprehend algebra

-a forum for dialogue about experiences with students' thinking in the classroom

• A replicable model for Mathematics Methods courses that facilitates preservice teachers' use of research on students' algebraic thinking, including:

-a minimum of 30 video clips of students' describing their algebraic thinking.

-a minimum of 10, hour-long teaching modules.

• An organized suite of applications for handhelds and tablets that help develop students' understanding of hard to comprehend algebraic concepts, identified by research, including:

-existing virtual manipulatives focused on algebra , a graphing calculator, spreadsheets, and access to science probes

-5-10 new virtual manipulatives addressing topics identified in research

-a classroom response system designed to use formative assessments identified in research

A consortium of four universities will restructure their mathematics teacher preparation programs to utilize the *Center* to:

• increase preservice teachers' *disposition* towards understanding and using students' algebraic thinking in their instructional decisions,

• increase preservice teachers' *knowledge* of algebra and the range of their students' thinking and misconceptions, and

• increase preservice teachers' *skills* in assessing students' thinking and implementing effective instruction addressing that thinking, including using technology based interventions.

• increase preservice teachers' students' passing rate in Algebra I courses by 50% and knowledge of algebra by 20%[4].

Formative evaluation and feedback loops will be integrated throughout the project to help us manage the project effectively and timely and refine resources and courses (see Figure 3, Appendix A). During Year 1, as Domain Teams synthesize research, once a month each team will review another team's work, looking for relevance to their work and providing feedback (see Figure 4, Appendix A). As resources are developed by the project they will be field tested by a team of two middle and two high school inservice teachers. Each member of the Core Team will be responsible for getting feedback from one teacher, based on that teacher's review of *Center* resources or trial implementation of technologies with their students. The Core Team will meet once a month throughout the project to process formative evaluation data, including data from surveys and tests as acquired, and revise project resources and courses in response.

Our primary goal is to increase preservice teachers' knowledge, disposition, and skills in using research on students' algebraic thinking. Our secondary goal is to increase preservice teachers' students' knowledge of algebra and passing rate in Algebra I courses. In order to determine whether project objectives are met and provide guidance about effective strategies suitable for replication in other settings our **Summative Evaluation** will include validated, existing tests and observation tools, case studies (including bi-weekly interviews of teachers and students), and *ATP*/ISTE developed surveys. ISTE will use a growth model with 30 preservice teacher classes across institutions, with pre- and post-test data as well as demographics as part of a multi-level model to control for preservice teacher and student level effects. Measures *ATP* will use to capture this data are described in the Project Evaluation Chart in Appendix A. The evaluation logic model is represented in Figure 5, Appendix A.

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[4] As measured by the algebra section of the Oregon Assessment of Knowledge and Skills (OAKS) and an existing, validated assessment of students' algebra knowledge.

Replicability and Dissemination Evaluation. At the end of the second year, Core Staff will design a survey of replicability authenticity that will identify the critical components of implementation. This self-administered review will help other institutions understand the faithfulness of their replicability efforts to the goals and objectives of the *ATP.* Dissemination will be measured by statistics on use of the *Center* web site and the number of institutions requesting support for replicating the Mathematics Methods courses model.

E. ADEQUACY OF THE RESOURCES

The *ATP* consortium is well prepared to implement this grant proposal to increase the students' success in algebra classrooms. We have a long history of collaborating to improve teacher education in Oregon. Our universities will provide the academic expertise of teacher educators, mathematicians (including an algebraist), and computer science faculty necessary to address the complex issues related to the development of algebraic thinking. Our combined facilities offer state of the art computing technologies and the computer science expertise to achieve maximum utilization of the potential power of handheld technology by adapting current applications and creating new applications for algebra learners. Each of our universities have strong computer services departments to support and maintain technology use.

Our business and organization partners of Shodor and Vernier will also provide the technical expertise and support for the creation of a suite of algebra applications for handhelds and tablets. Shodor will help develop virtual manipulatives while Vernier (using their own funding) will create interactivity between handhelds and science probes. Our teacher educators, public school mathematics teachers, and district level math specialists will provide the practical experience and insights into classroom dynamics to effectively develop and implement the intervention. Please see the letters in Appendix A for evidence of commitment of each partner to the *ATP.*

The financial resources to support this undertaking with multiple institutions are reasonable because each partner is critical to the success of the *ATP.* With such a vast research database, we will need the intellectual resources available from our respective institutions to process that literature and make it usable for preservice teachers. The Core Team will consist of a representative from each university partner. Our public/private partnership will expand the potential relevance to institutions across the country. Each of our institutions has a unique mode of delivery of Mathematics Methods courses, including different relations to student teaching, different timing in the program, and different lengths of courses. As *ATP* addresses these variables in restructuring our courses and development of resources, we will create a highly replicable model for institutions with varied approaches to teacher preparation.

The project costs are skewed toward the first year to reflect when resources are developed and refined. Most of those costs are devoted to personnel to process the database and design resources. A significant cost is the creation of virtual manipulatives for handhelds/web use that focus on specific, challenging areas of algebra for students to understand. We base those costs on Shodor's experience creating virtual manipulatives. Costs in Year 2 and 3 are primarily devoted to initial technology purchase for preservice teacher implementation, refining the program and resources, and data collection and analysis, particularly with the case studies. Willamette University will commit to budget $10,000 in additional funds to meet unforeseen grant needs.

In order to fund the *Center* continuously and the work of the monthly small group meetings following the conclusion of the grant, Dr. Rhine will take responsibility for managing the *Center.* He would get teacher education institutions and school districts to annually subscribe to the *Center* website as a resource for its preservice and inservice teachers for a small fee. Willamette University will commit to funding clerical work for the *Center* of approximately 5 hours per month and each university will sustain the tech libraries with a lab fee to students that will continually renew the library. In the case that more resources might be needed to do additional work such as creating new virtual manipulatives or videotape of students’ thinking, Dr. Rhine will write small grants to support that work.