Center for Algebraic Thinking

MODULE

Analysis of Change: Connecting graphs to real world data

BACKGROUND

Students frequently have difficulty creating graphs of real world situations. In this module candidates focus on connecting changing relationships to graphical representations.

1) **SET: Engage with a problem or problems that help teachers consider students' algebraic thinking (teachers' prior knowledge)**

Two people start at opposite corners of a room and walk towards each other. As they walk, they both slow down as they get closer to one another, pass, and then speed up as they get farther apart. This takes a total of eight seconds. The opposite corners of the room are 20 feet apart. Sketch a graph showing the relationship between time and the distance between the two people.*

Possible probing questions:
The problem states that they started at opposite corners. How is that represented on your graph?
How would your graph look different if both persons walked at a constant speed?
How is the fact that they are walking toward each other represented in your graph?
What difficulties do you think students have when they are deciding how to represent situations on a graph?

2) **STUDENTS: Watch video clips of students describing their thinking as they engage with problems (these video clips must still be developed/inserted)**

Imagine a person walking from a mark on the floor at one end of a room to the wall at the other end of the room and back to the mark on the floor. Draw a graph that would represent the distance you are from that mark over a period of time.**

(questions to ask students in the video)
Explain your graph.
What is represented on the x-axis? What is represented on the y-axis?
Where on your graph is the student furthest away from the mark?
Where on your graph is the student closest to the mark?
Does your graph show when the student isn't moving?

OR the Ramirez video (describe)?

After viewing the video: Discussion questions for pre-service teachers

- Where did students on the video struggle the most? Why do you think this particular piece is troublesome for students?
- How does the student thinking in the video compare to your own thought process when you were solving the problem about two people starting in opposite corners?
- How can teachers help students make connections between real-world situations and their representation on a graph?

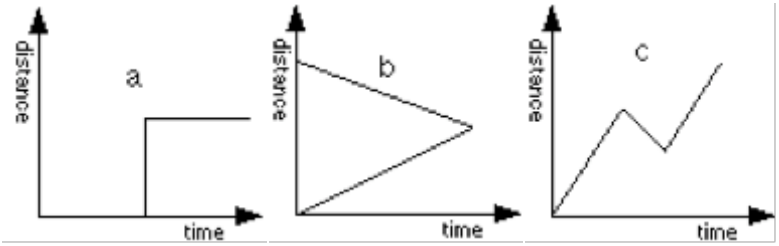
3) **RESEARCH: Examine/discuss research (encyclopedia entries)**

Research confirms that students can easily perform mechanical operations on graphs such as reading coordinates and finding slopes, but students have great difficulty describing the relationship between variables. For more information about this topic see encyclopedia entries on:

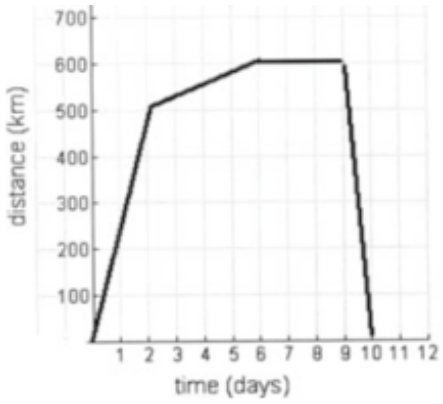
Constructing graphs of real world data
Graphs as a literal picture
Motion Graphs
Interpreting graphs

4) **ASSESSMENT: Consider assessments (formative assessment database)**

1. Which of the graphs below represent journeys? Describe what happens in each case. Why do you think that?



2. The Ramirez family's whole holiday is shown on the graph. The vertical axis shows the distance in kilometers away from home. The horizontal axis shows the time in days since the start of their trip.



a) During which days did the Ramirez family travel fastest?
b) They stayed with friends for a few days. Which days were these?
c) On average, how fast did the Ramirez family travel to get to their destination?

(Herbert & Pierce 2008)

3. Sarah, Rivka, Rachel and Leah discussed the question of whether or not their success on tests is related to the amount of time that they prepare for tests.

A. Sarah claimed that the more that she studies, the better her grades are. Please construct a graph that represents Sarah's claim.

B. Rivka argued that no matter how long she studies, she always gets the same grade. Please construct a graph that represents Rivka's claim.

C. Rachel, however, said that when she studies up to three hours, the longer she students the better her grades; but, beyond three hours, she becomes tired and her grades are lower. Please construct a graph that represents Rachel's claim.

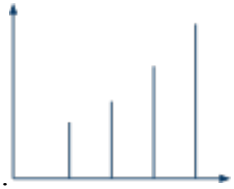
D. Leah confessed that for her, generally, when she studies more, her grades descrease. Please construct a graph that represents Leah's claim. (Mevarech, 1997)

5) **SUGGESTED TEACHING STRATEGIES: Consider strategies based on research (including apps)**

Technology can be a powerful way for students to explore situations and their graphical representations. It allows students to concentrate on analyzing the relationships instead of focusing on constructing the graphs.: The following app can be used to emphasize how the same situation can have multiple graphs depending on which variables are being considered.
http://www2.edc.org/edc-research/curriki/ROLE/lc/sessions/session6/bike.htm
Using a Calculator Based Ranger (CBR) there are multiple activities that allow students to make connections between walking rates and graphs:
http://education.ti.com/calculators/downloads/US/Activities/Search/Subject?s=5022&sa=5022&t=5030&d=1

Use Graphs without numbers so students must look more closely at the relationships instead of the numbers. (Shell Centre)

10 year old children who have had little or no training in graphing can represent quantities such as the speed of a car fairly easily if they use vertical lines to represent the speed when the distance is represented on the x axis. This method originates back to Oresme (1323-1382) and suggests that using vertical lines would be a useful starting place when teaching students how to draw graphs. See below for example.



Students should collect their own data, construct tables, and then graphs. There should be a discussion about how they connect the graph to the real world situation so that teachers have a chance to see student reasoning. Ask questions such as: Where on your graph does it show where the student turns around and heads back? Is the speed at which the student is traveling reflected on your graph?

Creating a table of values before making a graph is a good way for students to see patterns, but sometimes a table may lead to overly procedural thinking (students can become comfortable with finding number patterns and 'lose' the true relationship between the variables). Take into account students' previous background knowledge of a situation. See Encyclopedia entry "Student Background Knowledge and Graphing."

Students know a great deal about representations apart from formal mathematical instruction and are able to invent new forms of representation. Asking students to invent their own representations may give us greater insight into their overall understanding of representation. The teacher must be prepared to respond flexibly to a wide range of student inventions and ideas. It requires, at a minimum, that the teacher know some of what to expect as the class proceeds: what kinds of inventions students might offer, which inventions offer productive avenues to follow, and how to help guide student work in these productive directions. (Sherin 2000, p. 408)

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