Center for Algebraic Thinking

# MODULE

## *Variables & Expressions: Representation*

**BACKGROUND**
There is a developmental progression in understanding the meaning of variables. A key concept is understanding what variables can represent and how that is influenced by context.

1) **SET: Engage with a problem or problems that help teachers consider students' algebraic thinking (teachers' prior knowledge)**
A problem/scenario in which teachers need to struggle with the concept of representation.
Discuss the word "Represent". How is the word used the same across situations within a context vs. different across contexts?
**Represent**
-equation/data graphically/pictorially
-political/race/gender
-personal meaningful
-analogy (democracy is like a fruit salad/making sausage)
-map
-cultural interpretation
-results of a survey

Parallels with variables:
• variables can change across context but stay the same within context
(two equations: $4x + 4 = 9$ and $3x - 2 = 3$; $5x + 34x = 4 - 3x$)

2) **STUDENTS: Watch video clips of students describing their thinking as they engage with problems**
• View Girl 4, Girl 3, and Boy 3 for the problem:
When is the following true? Always, sometimes or never: $h + 2 = h$

• View Boy 3 and Boy 4 for the problem:
Which is larger, $2n$ or $n + 2$?
Girl 1: I don't know because I don't know n
Girl 2: correct
Boy 2: doesn't understand n within context

What do you learn from what you are hearing or seeing regarding students' thinking?

Consider contexts, operations with variables and meaning of variable, number sense, different answers in different cases (when $n<2$, $n + 2$ is greater; when $n=2$, they are the same; when $n>2$, $2n$ is larger))

Other videos to view for this module:
•Is this equation always, sometimes, or never true? a + b = b Why?
-do the two b's have the same value?

•If e + f = 8, what does e + f + g = ? How do you know?

•If you have "2n + 5"
Can "n" stand for "4"?
"37"?
"-149"?
"3r + 2"?

• Are these two equations equal or equivalent to each other? Why or Why not?
3x = 5 + 4 3 + x = 5 + 4

3) **<u>RESEARCH</u>: Examine/discuss research (encyclopedia entries)**
Introduce entries on Progression of Understanding Variables.
Review "Variables: Representation" entry.

*Letter Evaluated entry*

| **PROBLEM** | **RESPONSE** |
|---|---|
| 1. If e + f = 8, e + f + g = ? | 15 (2%)<br>12 (26%)<br>8g (3%)<br>9 (6%)<br>Correct, 8 + g (41%)* |

Highlight key findings from the research:

4) **<u>ASSESSMENT</u>: Consider assessments (Formative Assessment Database)**
•Which is larger, 2n or n + 2?

•Is this equation always, sometimes, or never true? a + b = b Why?

•If e + f = 8, what does e + f + g = ? How do you know?

•If you have "n + 5"
Can "n" stand for "4"? Can "n" stand for "37"? Can "n" stand for "-149"? Can "n" stand for "3r + 2"?

• Are these two equations equal or equivalent to each other? Why or Why not?
3x = 5 + 4 3 + x = 5 + 4

5) **<u>SUGGESTIONS FOR TEACHING</u>: Consider strategies based on research (including apps)**
Highlight encyclopedia entries teaching strategies.

Examine spreadsheets as a tool for developing the concept of variable.
-A cell representing the variable and its potential to be anything.

6) **<u>Did the preservice teachers understand? How do you know? Evidence</u>**

**<u>REFERENCES</u>**

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